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Turbulence measurements in a natural convection boundary layer along a vertical flat plate

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Abstract—A turbulent natural convection boundary layer, the structure of which still remains a matter of conjecture for want of reliable measurement, is investigated with the V-shaped hot-wire technique. The reliability of Reynolds stress and turbulent heat flux measurements is verified by the excellent agreement with the indirect measurements estimated by integrating momentum and thermal energy equations with measured mean velocity and mean temperature. Turbulent quantities clarified quantitatively in the present study indicate that the natural convection boundary layer has a unique turbulent structure rarely seen in other turbulent boundary layers.

1. INTRODUCTION

THE INVESTIGATION of a turbulent natural convection boundary layer along a vertical flat plate is very important not only to clarify the heat transfer mechanism but also to elucidate the basic structure of turbulent transport phenomena. In a previous paper [1], we conducted measurements of mean velocity and mean temperature profiles, heat transfer rate and wall shear stress for the turbulent natural convection boundary layer in air. As a result, we found some quantitatively distinct characteristics attendant on natural convection, which were interesting and rarely encountered in other turbulent boundary layers. To evaluate these characteristics and clarify still further the turbulent structure of natural convection, detailed measurements concerning turbulent quantities must also be carried out.

There are some precedents in measuring velocity and temperature fluctuations in the turbulent natural convection boundary layer in air. Smith [2] and Cheesewright and Doan [3] conducted hot-wire measurements, for example. Miyamoto and Okayama [4], Miyamoto *et al.* [5] and Cheesewright and Ierokipiotis [6, 7] also measured turbulent quantities using LDV. However, because of the difficulty in obtaining the measurements, only a few results were reliable and many unknown parts still remain. In particular, the turbulent characteristics near the wall, which are important in relation to heat transfer, have not been clarified even in quality and are in a state of supposition.

In the present study, turbulent quantities of velocity and temperature are investigated using a probe, comprising two V-shaped hot wires [8–10] advantageous to the measurement near the wall and a cold wire. For measurements comparable to those of the previous paper [1], the measurement accuracies are examined by comparison. Also, it is demonstrated that the direct measurements of Reynolds stress and turbulent heat flux, which are very important quantities in terms of turbulent structure, have quantitatively high reliability. They are compared with the calculated values obtained by integrating the momentum and thermal energy equations for mean flow, substituting mean velocity and mean temperature profiles into the equations. These results clearly indicate that the natural convection boundary layer possesses interesting characteristics in the turbulent structure, which are rarely seen in other turbulent boundary layers.

2. EXPERIMENTAL APPARATUS AND PROCEDURE

The apparatus used for natural-convection measurements was the same as in the previous study [1]. The heated surface consisted of a 4 m high, 1 m wide and 2 mm thick copper plate and was maintained at a uniform temperature by electric heaters equipped on the back of the plate. To produce a stable two-dimensional turbulent boundary layer, the apparatus was devised so that the change of ambient fluid temperature in the height direction became as small as possible, paying attention to the condition of ambient fluid to be entrained into the boundary layer.

For the measurement of fluid velocities, we used a V-shaped hot wire, constructed by symmetrically bending a 3.1 μm diameter and 1.5 mm long tungsten wire at the center. The V-shaped hot wire is most effective for measuring turbulent quantities near the wall and does not receive the influence of prongs, as compared to the inclined hot wire usually used. Characteristics of the V-shaped hot wire in high Reynolds number flows have been established by Hishida and Nagano [11, 12] and highly accurate measurements of turbulent quantities using the hot wire have been conducted in the studies cited [13, 14]. However, the characteristics of the hot wire in a low-velocity

NOMENCLATURE

- c_p specific heat at constant pressure
- $F(u), F(v), F(t)$ flatness factors of u, v and t
- Gr_x local Grashof number, $g\beta\Delta T_w x^3/\nu^2$
- Gr_x^* modified local Grashof number, $g\beta q_w x^4/\lambda\nu^2$
- g gravitational acceleration
- Pr_t turbulent Prandtl number, equation $(6)_3$
- $P(u/\sqrt{\overline{u^2}}), P(v/\sqrt{\overline{v^2}}), P(t/\sqrt{\overline{t^2}})$ probability density functions of $u/\sqrt{\overline{u^2}}$, $v/\sqrt{\overline{v^2}}$ and $t/\sqrt{\overline{t^2}}$
- q_w wall heat flux
- R_{uv}, R_{vt}, R_{ut} cross-correlation coefficients, equations (5)
- $S(u), S(v), S(t)$ skewness factors of u, v and t
- T mean fluid temperature
- T^+ dimensionless temperature, $(T_w - T)/t_*$
- ΔT_w temperature difference, $T_w - T_\infty$
- t temperature fluctuation
- t_* friction temperature, $q_w/\rho c_p u_*$
- U mean streamwise velocity
- U_m maximum velocity along flat plate
- U^+ dimensionless velocity, U/u_*
- u streamwise velocity fluctuation
- u_* friction velocity, $\sqrt{(\tau_w/\rho)}$
- V mean transverse velocity (perpendicular to flat plate)
- V^+ dimensionless velocity, V/u_*
- v transverse velocity fluctuation
- w spanwise velocity fluctuation (parallel to flat plate)
- x distance from the leading edge of flat plate
- y distance perpendicular to flat plate
- y_m maximum velocity location
- y^+ dimensionless distance from wall, $u_* y/\nu$
- z spanwise distance from the center line of flat plate.

Greek symbols

- α thermal diffusivity
- β coefficient of volume expansion, $1/T_\infty$
- $\varepsilon_m, \varepsilon_h$ eddy diffusivities for momentum and heat, equations (6)
- ζ dimensionless transverse coordinate, $-y(\partial\theta/\partial y)_{y=0}$
- θ dimensionless temperature, $(T - T_\infty)/\Delta T_w$
- λ thermal conductivity
- ν kinematic viscosity
- ρ density
- $\sigma_{uv}, \sigma_{vt}, \sigma_{ut}$ fluctuating amplitudes of uv, vt and ut (standard deviations), equations (4)
- τ_w wall shear stress.

Superscript

- $\overline{}$ time averaged quantities.

Subscripts

- f film temperature
- w wall condition
- ∞ ambient condition.

turbulence field such as natural convection differ from those in high Reynolds number flows. Prior to the present experiment, therefore, the characteristics at low velocities and the measurement accuracy of the V-shaped hot wire were examined, and it was confirmed that this technique was very effective even for low-velocity turbulence measurement, as reported in detail in refs. [8–10].

The two V-shaped hot wires were combined in a ×-array as shown in Fig. 1, and the velocity fluctuations u and v were measured in the height and normal directions of the heated surface, respectively. A cold wire of 3.1 μm diameter and 3.5 mm ($\simeq 14\nu/u_*$) long tungsten wire was located upstream of the hot wires for temperature compensation, and the hot-wire outputs were separated into velocity and temperature signals using the technique of Hishida and Nagano [15]. Special care was exerted so that the whole probe was as small as possible so as not to compromise spatial resolution, without the interference of velocity and thermal fields between the fine wires. Moreover, to minimize the measurement error caused by the spatial difference between thermal fields around the hot wires and the cold wire, the temperature compensations of the hot wires were made more precisely by delaying temperature signals in the same manner as in the previous paper [1].

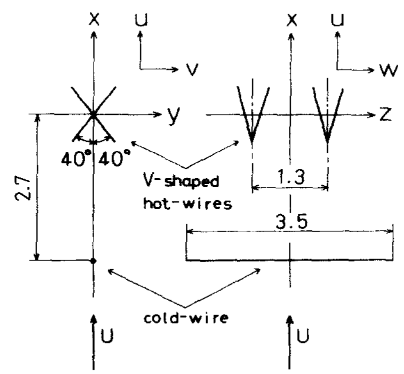

FIG. 1. Arrangement of V-shaped hot wires and cold wire.

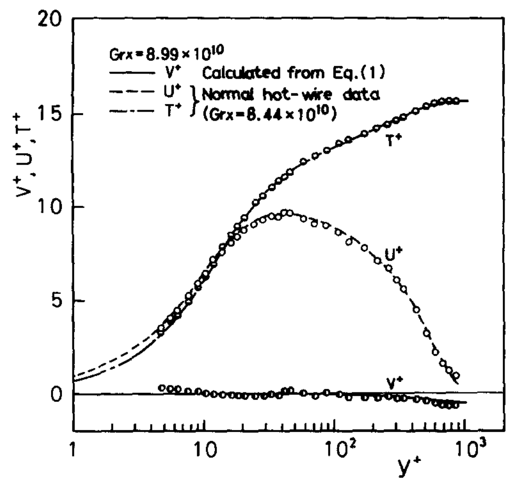

FIG. 2. Mean velocity and mean temperature profiles.

Measured data were recorded with a data-recorder for 1–5 min and processed by computer (FACOM-M382) after analog to digital conversion at a sampling frequency of 400 Hz.

Turbulence measurement was carried out in the turbulent boundary layer at a height of 2.54 m ($Gr_x = 8.99 \times 10^{10}$) from the leading edge of the heated surface. In the experiment, surface temperature T_w was 60.6°C and ambient fluid temperature T_∞ was 15.3°C, which increased at the rate of about 0.6°C m^{-1} in the height direction.

Prior to the measurement for the turbulent boundary layer, velocity U, V and temperature T in the laminar boundary layer were obtained with the V-shaped hot-wire probe. All profiles agreed very well with the theoretical values for the laminar boundary layer [1]. Thus, it was confirmed that the use of the V-shaped hot wire at low velocities served to yield adequate results.

Physical properties were evaluated with film temperature $T_f = (T_w + T_\infty)/2$ except the body expansion coefficient $\beta = 1/T_\infty$.

3. MEAN VELOCITY AND MEAN TEMPERATURE PROFILES

Mean velocity and mean temperature profiles, normalized by the friction velocity u_* and the friction temperature t_*, are shown in Fig. 2 as a function of y^+. In evaluating u_* and t_*, the wall shear stress τ_w and the wall heat flux q_w were obtained from the near-wall profiles of velocity and temperature [1]. For U^+ and T^+, the measurements using a normal hot wire and a cold wire presented in the previous paper [1] are indicated by the broken line and the chain line, respectively. The solid line indicates the value of V^+ estimated by substituting the U measurements [1] into the following integral equation of continuity:

$$V = -\int_0^y \frac{\partial U}{\partial x} \mathrm{d}y. \qquad (1)$$

The agreements for each profile are excellent over the entire region in the boundary layer. In natural convection, the value of V in the outer region beyond the maximum velocity location is slightly negative due to the entrainment of ambient fluid.

Generally, the hot-wire measurement very near the wall is difficult since velocity and thermal fields around the hot wire are subject to the wall-proximity effects. For the measurements of natural convection, the effects were observed in the range $y \leqslant 1$ mm ($y^+ \leqslant 4$). Such unreliable data are excluded in Fig. 2 and also in the results presented later. As in the previous paper [1], we refer to the region extending from the maximum velocity location y_m to the edge of the boundary layer as the outer layer, and to the region between y_m and the heated surface as the inner layer.

4. VELOCITY AND TEMPERATURE FLUCTUATIONS

4.1. *Waveforms of fluctuations*

Waveforms of the velocity fluctuations u and v and temperature fluctuation t are shown in Fig. 3. In the waveforms of velocity fluctuations at $y^+ = 7.6$ near the wall, the laminar-like flow patterns of extremely small fluctuation are observed. As to the waveforms of temperature fluctuation, a fairly low temperature is observed occasionally. At $y^+ = 16.1$, somewhat apart from the wall, the amplitude of the velocity fluctuation u and the temperature fluctuation t markedly increase, whereas the amplitude of the fluctuation v does not. The large amplitude temperature fluctuations ranging from wall to ambient temperatures are seen here in the temperature waveform, and so the intensity of the temperature fluctuation becomes maximum near this location, as described later. Although the region $5 < y^+ < 30$ in the turbulent forced convection boundary layer is equivalent to the buffer layer where the coherent turbulent structure dominates and the strong relation between the waveforms of velocity and temperature fluctuations is seen [14], such behavior is not so evident between the waveforms in natural convection. Near the maximum velocity location ($y^+ = 41.3$), fluctuating components of high frequency increase in all waveforms; but the maximum amplitudes of velocity fluctuations u and v do not differ so much from those in the inner layer of $y^+ = 16.1$. For the temperature fluctuation, a high temperature irregularly appears in the waveform contrary to that observed at $y^+ = 7.6$ near the wall. At $y^+ = 256.3$ in the outer layer, high frequency components are seen to overlap large amplitude velocity fluctuations of low frequency. The intensities of velocity fluctuations u and v, as described later, become maximum at this location. On the other hand, the waveform of the temperature fluctuation is fairly intermittent and the fluctuating amplitude becomes very small. As the edge of the boundary layer is approached (i.e. $y^+ = 509.1$), all waveforms become

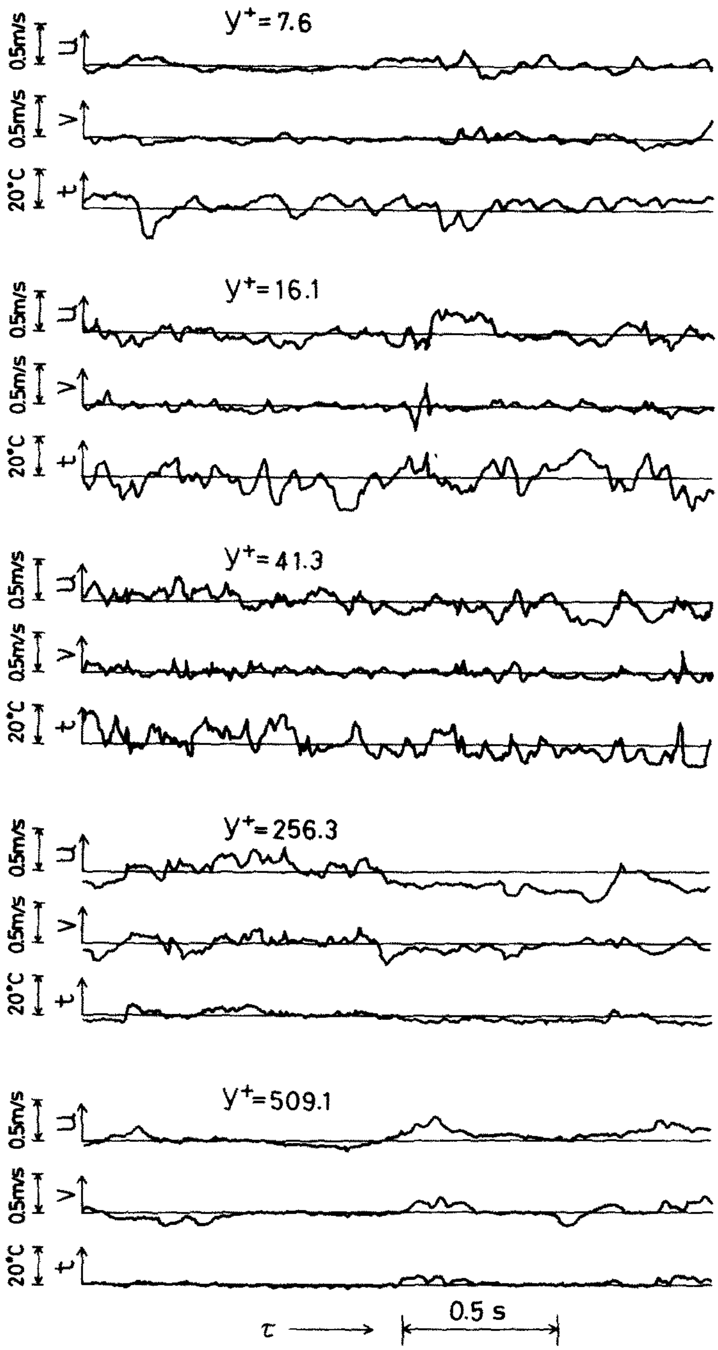

FIG. 3. Simultaneous traces of velocity and temperature fluctuations.

intermittent and the amplitudes of the fluctuations are also small.

4.2. *Intensities of velocity and temperature fluctuations*

The intensity distributions of velocity and temperature fluctuations are shown in Fig. 4 in the form of $\sqrt{\overline{u^2}}/u_*$, $\sqrt{\overline{v^2}}/u_*$ and $\sqrt{\overline{t^2}}/t_*$. The measurements of $\sqrt{\overline{u^2}}$ and $\sqrt{\overline{t^2}}$ using the normal hot-wire probe [1], shown in the figure, agree very well with those using the V-shaped hot-wire probe. The intensity $\sqrt{\overline{v^2}}$ is smaller than $\sqrt{\overline{u^2}}$ in the entire boundary layer region and becomes a maximum at almost the same location in the outer layer ($y^+ \simeq 250$) as the $\sqrt{\overline{u^2}}$ maximum. There have been few measurements of velocity fluctuation v in natural convection except for those reported by Smith [2] and Miyamoto *et al.* [5].

A comparison of the intensity $\sqrt{\overline{v^2}}$ with other measurements is shown in Fig. 5 in the relation between $\sqrt{\overline{v^2}}/U_m$ and $\zeta = -y(\partial\theta/\partial y)_{y=0}$. The data obtained by Smith [2] with a ×-wire are relatively limited in the narrow range of the outer layer, and the measurement accuracy is uncertain. The result

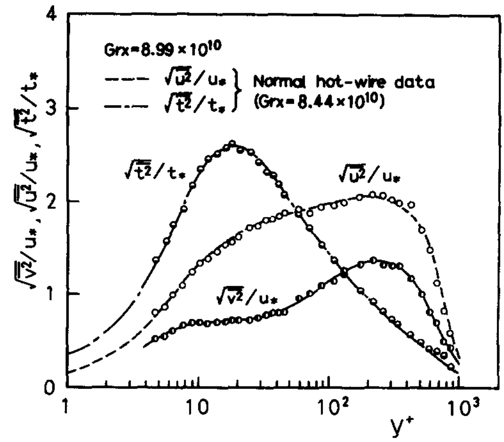

FIG. 4. Intensity distributions of velocity and temperature fluctuations.

of Miyamoto *et al.* [5], who carried out an LDV measurement with uniform surface heat flux, resembles the present result in the form of distribution; their maximum intensity location almost agrees with that in the present result. However, a somewhat different trend is seen near the wall. In this region, although $\sqrt{\overline{v^2}}$ is only half the $\sqrt{\overline{u^2}}$ in the present experiment, the $\sqrt{\overline{v^2}}$ value measured by Miyamoto *et al.* [5] is remarkably smaller than the $\sqrt{\overline{u^2}}$ value, becoming about 1/6–1/9 of the $\sqrt{\overline{u^2}}$ value in the case of large Gr_x. Such a decreasing behavior of $\sqrt{\overline{v^2}}$ relative to the development of the boundary layer is unnatural, while $\sqrt{\overline{u^2}}$ increases with Gr_x.

Comparisons of $\sqrt{\overline{u^2}}$ and $\sqrt{\overline{t^2}}$ intensities with other measurements coincide with those in the previous paper [1] and are therefore omitted in the present report.

5. REYNOLDS STRESS AND TURBULENT HEAT FLUXES

5.1. *Indirect measurements of $\overline{uv}$ and $\overline{vt}$*

Generally, Reynolds stress $\overline{uv}$ and turbulent heat flux $\overline{vt}$ in the normal direction to the wall are important factors controlling the characteristics of the turbulent boundary layer. These quantities can be obtained by performing a direct measurement with a hot-wire/cold-wire arrangement and/or an indirect measurement based on the calculation with measured mean velocity and mean temperature profiles. Although the direct measurement obtaining data in time series is desirable for the various analyses of turbulent quantities, the indirect measurement is also useful to examine the reliability of direct measurement. The indirect measurement is described by the following procedure. Here, for the sake of simplicity, the Boussinesq approximation is used and physical properties are assumed to be constant.

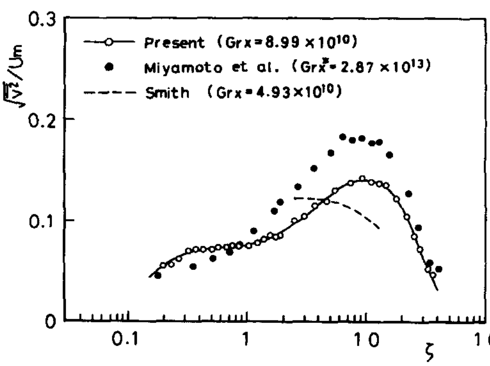

FIG. 5. Comparison of intensities of the v fluctuation.

The integral equations of momentum and thermal energy for mean flow in the turbulent natural convection boundary layer are expressed as follows:

$$\overline{uv} = \underbrace{-\int_0^y \left(U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y}\right)\mathrm{d}y}_{\text{AT}} + \underbrace{\nu\int_0^y \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right)\mathrm{d}y}_{\text{VT}} \underbrace{+ g\beta\int_0^y (T - T_\infty)\,\mathrm{d}y}_{\text{BT}} \underbrace{- \int_0^y \frac{\partial}{\partial x}(\overline{u^2} - \overline{v^2})\,\mathrm{d}y}_{\text{FT}} \quad (2)$$

$$\overline{vt} = \underbrace{-\int_0^y \left(U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y}\right)\mathrm{d}y}_{\text{AT}} + \underbrace{\alpha\int_0^y \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)\mathrm{d}y}_{\text{CT}} \underbrace{- \int_0^y \frac{\partial \overline{ut}}{\partial x}\mathrm{d}y}_{\text{FT}}. \quad (3)$$

Although the terms of $\nu\,\partial^2 U/\partial x^2$ and $\alpha\,\partial^2 T/\partial x^2$ for mean flow and $\partial(\overline{u^2} - \overline{v^2})/\partial x$ and $\partial\overline{ut}/\partial x$ for fluctuating components in equations (2) and (3) can be neglected conventionally for high Reynolds number boundary layers [16, 17], these terms are taken into account here. But the term $\partial\overline{v^2}/\partial x$ may be omitted because $\partial\overline{v^2}/\partial x \ll \partial\overline{u^2}/\partial x$. By substituting the measurements U, T, $\overline{u^2}$ and $\overline{ut}$ presented in the previous paper [1] into the above equations and performing the integrations, the distributions of Reynolds stress $\overline{uv}$ and turbulent heat flux $\overline{vt}$ can be obtained. For the value of V in equations, it is sufficient to use the continuity equation (1) described before.

The calculated $\overline{uv}$ and values of each term in equation (2) are shown in Fig. 6(a), being normalized by u_*^2. In forced convection, Reynolds stress is connected so closely with the mean velocity gradient as $\overline{uv} < 0$ when $\partial U/\partial y > 0$ and $\overline{uv} > 0$ when $\partial U/\partial y < 0$. Therefore, the $\overline{uv}$ value usually becomes negative in the near-wall region of $\partial U/\partial y > 0$. However, Reynolds stress near the wall in natural convection is almost

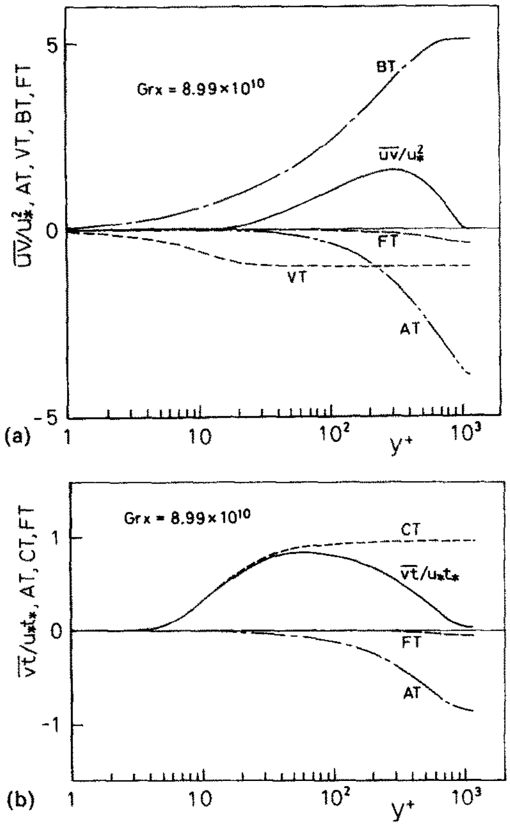

FIG. 6. Indirect measurements of Reynolds stress and turbulent heat flux. (a) $\overline{uv}$ and values of each term in equation (2): AT, advection term; VT, viscous diffusion term; BT, buoyancy term; FT, fluctuation term. (b) $\overline{vt}$ and values of each term in equation (3): AT, advection term; CT, thermal conduction term; FT, fluctuation term.

zero in spite of $\partial U/\partial y > 0$, as seen in Fig. 6(a), because the buoyancy term (BT) cancels out the viscous diffusion term (VT). As the maximum velocity location is approached, the viscous diffusion term (VT) becomes almost constant and so $\overline{uv}$ increases in the positive value with an increase of the buoyancy term (BT). Consequently, the maximum velocity location (where $\partial U/\partial y = 0$) does not coincide with the location of $\overline{uv} = 0$. The maximum of $\overline{uv}$ occurs at the location where the intensities of velocity fluctuations become maximum in the outer layer, as shown in Fig. 4. Thus, the outer layer has the same characteristic as the forced convection boundary layer, because $\partial U/\partial y < 0$ and $\overline{uv} > 0$. At the outer edge of the boundary layer, the values of each term in equation (2) are counterbalanced and $\overline{uv}$ becomes roughly zero. Thus, the measured values used for the calculation are highly reliable. For instance, the measured value of wall shear stress presented in the previous paper [1] agrees with that estimated from the momentum balance to within 10% (including an error due to the assumption of constant properties in the calculation). The contribution of the $\nu\,\partial^2 U/\partial x^2$ term is so small as to be negligible. Although the $\partial \overline{u^2}/\partial x$ term is also small, it is necessary to take into account this term for the momentum balance as shown in Fig. 6(a).

Figure 6(b) shows the values of each term in equation (3) and turbulent heat flux $\overline{vt}$ obtained from equation (3), being normalized by $u_* t_*$. Near the wall, the advection term (AT) is small and so $\overline{vt}$ increases largely in accordance with the thermal conduction term (CT). Temperature variance $\overline{t^2}$ is generated by $-2\overline{vt}(\partial T/\partial y)$. Therefore, the maximum intensity of the temperature fluctuation occurs at the location ($y^+ \simeq 15$), where this quantity takes the maximum, as seen in Fig. 4. The $\overline{vt}$ value becomes maximum with the increase of the advection term (AT) in the vicinity of the maximum velocity location, and then decreases toward the outer edge of the boundary layer. This $\overline{vt}$ distribution agrees relatively with the measured result for forced convection [14]. At the outer edge of the boundary layer, $\overline{vt}$ approaches zero as the values of each term are balanced. The term $\alpha\,\partial^2 T/\partial x^2$ is so small as to be eventually negligible. Although the $\partial \overline{ut}/\partial x$ term for fluctuating components is also small, it must be taken into account for the thermal energy balance, since its integral value up to the outer edge of the boundary layer is about 5% of the integral value of the thermal conduction term (CT). Then, the measured wall heat flux [1] agrees with that estimated from the thermal energy balance to within 10%, like the case of wall shear stress.

5.2. *Directly measured result*

The distributions of Reynolds stress $\overline{uv}$ and turbulent heat flux $\overline{vt}$ obtained with the V-shaped hot-wire probe are shown in Fig. 7. The solid line and the broken line in the figure indicate the respective values of $\overline{uv}$ and $\overline{vt}$, calculated from equations (2) and (3). Both measured $\overline{uv}$ and $\overline{vt}$ agree very well with the calculated values over the entire boundary layer region, although they tend to be slightly smaller than the calculated values for $y^+ > 300$ in the outer layer. Since velocity fluctuations increase relative to the mean velocity in the outer layer, the case often occurs in which hot-wire anemometry using a ×-array probe cannot identify the flow direction (rectification) [10]. The trifling difference between the measured and calculated values for $y^+ > 300$ might be caused mainly by the rectification. Also shown in Fig. 7 is the streamwise turbulent heat flux $\overline{ut}$ measured simultaneously and agreeing very well over the whole boundary layer region with the measured result obtained by using the normal hot-wire probe [1]. From these comparisons, the present direct measurements for Reynolds stress and turbulent heat fluxes are judged to be sufficiently reliable.

When tracing the instantaneous Reynolds stress uv and turbulent heat fluxes vt and ut, their waveforms include considerably intermittent fluctuations. The distributions of Reynolds stress and turbulent heat fluxes in Fig. 7 were obtained as the time-averaged values of these instantaneous fluctuations. Therefore, the fluctuating amplitudes around the time-averaged values (standard deviations), as defined by the following equations for instantaneous values of Reyn-

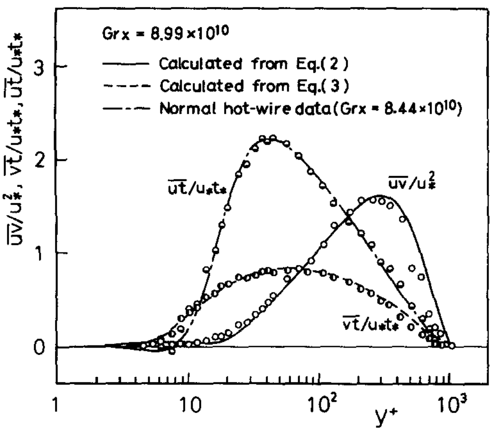

FIG. 7. Direct measurements of Reynolds stress and turbulent heat fluxes.

olds stress and turbulent heat fluxes, were investigated and shown in Fig. 8:

$$\sigma_{uv} = [\overline{(uv-\overline{uv})^2}]^{1/2}/u_*^2$$

$$\sigma_{vt} = [\overline{(vt-\overline{vt})^2}]^{1/2}/u_*t_*$$

$$\sigma_{ut} = [\overline{(ut-\overline{ut})^2}]^{1/2}/u_*t_*. \quad (4)$$

Reynolds stress $\overline{uv}$ has a value close to zero near the wall, as shown in Fig. 7. The corresponding amplitude of the uv fluctuation near the wall also becomes small but not negligible. The maximum value location of the fluctuating amplitude is observed in the outer layer and roughly coincides with the location where $\sqrt{\overline{u^2}}$, $\sqrt{\overline{v^2}}$ and $\overline{uv}$ become maximum in common. This σ_{uv} distribution bears a resemblance to the $\sqrt{\overline{v^2}}$ distribution shown in Fig. 4. Therefore, it is evident that the generation of Reynolds stress relies strongly on the behavior of the velocity fluctuation v. Turbulent heat fluxes $\overline{vt}$ and $\overline{ut}$ become maximum in the vicinity of the maximum velocity location, as seen in Fig. 7. But those fluctuating amplitudes take the maximums at a location closer to the wall. Hence, it is expected that the fluctuations of vt and ut depend heavily on the amplitude of the t fluctuation (which is maximum at $y^+ \simeq 15$). Although there appears to be a $\overline{ut} \leqslant \overline{vt}$

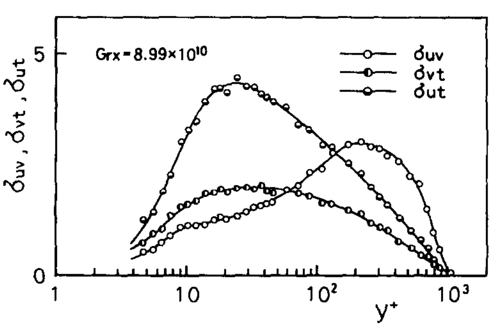

FIG. 8. Fluctuating amplitudes of uv, vt and ut (standard deviations).

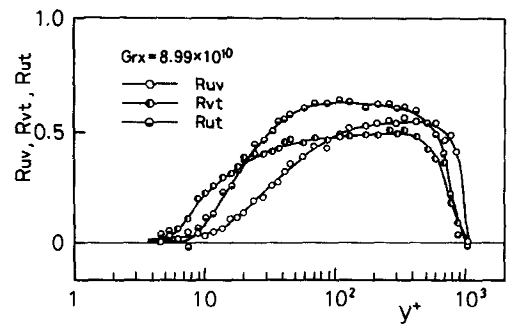

FIG. 9. Cross-correlation coefficients between velocity and temperature fluctuations.

trend near the wall, the fluctuating amplitude of ut exceeds that of vt in the overall boundary layer region, because of the large amplitude of the u fluctuation.

The cross-correlation coefficients between velocity and temperature fluctuations defined by the following equations are presented in Fig. 9:

$$R_{uv} = \overline{uv}/\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}$$

$$R_{vt} = \overline{vt}/\sqrt{\overline{v^2}}\sqrt{\overline{t^2}}$$

$$R_{ut} = \overline{ut}/\sqrt{\overline{u^2}}\sqrt{\overline{t^2}}. \quad (5)$$

The coefficient R_{uv}, which is almost zero near the wall, increases beyond $y^+ = 10$ and takes a constant value of roughly 0.5–0.55 in the region $100 < y^+ < 700$. On the other hand, R_{vt} already increases very near the wall and has a constant value of about 0.45–0.5 in the region from the maximum velocity location to the outer layer. Although R_{ut} shows a value smaller than R_{vt} near the wall, this correlation coefficient has a constant value of 0.65 greater than R_{vt} over a wide range from the maximum velocity location to the outer layer.

5.3. *Comparisons with other measurements*

Figure 10(a) compares Reynolds stress $\overline{uv}$ in the present experiment and other measurements. The measurement of Smith [2] was limited to a narrow measuring range and differed considerably from the present result, both quantitatively and qualitatively. Although the result of Miyamoto *et al.* [5] using an LDV tended to be somewhat small near the wall and in the outer layer, it agrees rather well with the present experiment for the most part of the boundary layer. The result of Cheesewright and Ierokipiotis [7] was not obtained directly, but was an indirect result obtained by integrating a momentum equation with the measurements of mean velocity and mean temperature using an LDV and a thermocouple. Also, in the present study, the distribution of Reynolds stress was estimated by means of a similar method to verify the direct measurement, as mentioned above. However, the accuracy of the estimation depends greatly on the measurement accuracies of basic mean velocity and temperature profiles. The Cheesewright and Ierokipiotis result, although showing the same

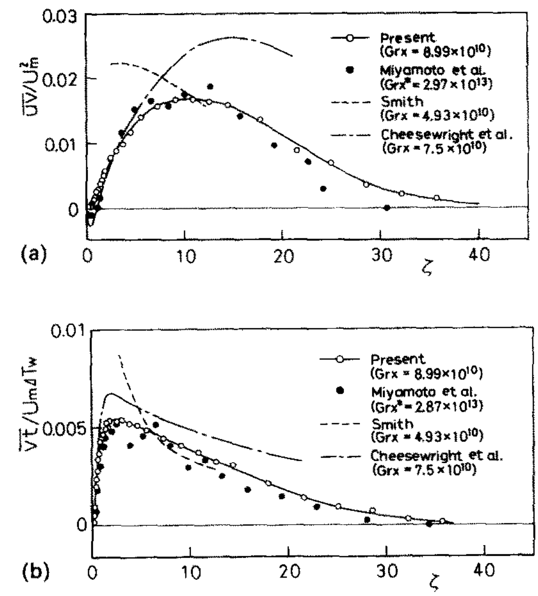

FIG. 10. Comparison of Reynolds stress and turbulent heat flux: (a) Reynolds stress $\overline{uv}$; (b) turbulent heat flux $\overline{vt}$.

trend as the present result and Miyamoto *et al.* [5], becomes large in the negative direction near the wall but large in the positive direction in the outer layer. Therefore, their accuracy leaves something to be desired.

A comparison of the measured results concerning turbulent heat flux $\overline{vt}$ is presented in Fig. 10(b). Although there is a different tendency between the result of Smith [2] and the present experiment, the measurement of Miyamoto *et al.* [5] virtually agrees with the present result over the entire boundary layer region. The measurement of Cheesewright and Ierokipiotis [7] is an indirect result obtained by substituting mean velocity and mean temperature profiles into the integral equation of thermal energy in the same manner as the estimation of $\overline{uv}$. Their result reveals a trend in accord with the present experiment qualitatively but a somewhat large value.

The configuration of the mean velocity profile in the natural convection boundary layer is similar to that of a two-dimensional wall jet and thermal plume along a vertical plate. In the case of the two-dimensional wall jet [18, 19], the maximum velocity location still disagrees with the location where the Reynolds stress is zero, but the Reynolds stress near the wall obviously has a negative value. Moreover, a logarithmic profile exists in the universal velocity profile $U^+ = f(y^+)$ and so there are considerable resemblances with forced convection. In the thermal plume along a vertical wall [20], $\overline{uv}$ has a positive constant value in the wide region from the near wall to the outer layer. These turbulence fields belong to a system in which the energy of velocity fluctuations diffuses and decays in the flow direction. Therefore, it is readily conjectured that such structures near the wall are considerably different from that of the turbulence field such as the natural convection boundary layer along a vertical plate, in which the buoyant energy is continuously supplied from the wall in the flow direction. For the turbulence structure of the outer layer, there are some qualitative resemblances between natural convection and a wall jet. For example, the maximum Reynolds stress occurs in the region from the maximum velocity location to the location at which mean velocity becomes one half of the maximum value and where the intensities of u and v also become maximum [18, 19, 21].

5.4. *Eddy diffusivities for momentum and heat and turbulent Prandtl number*

The distributions of eddy diffusivities ε_m and ε_h for momentum and heat and turbulent Prandtl number Pr_t, defined by the following equations are shown in Fig. 11:

$$-\overline{uv} = \varepsilon_m \, \partial U/\partial y$$
$$-\overline{vt} = \varepsilon_h \, \partial T/\partial y$$
$$Pr_t = \varepsilon_m/\varepsilon_h. \qquad (6)$$

When considering the structure of turbulent heat transfer, these quantities are essentially important but have not been shown until now for a turbulent natural convection boundary layer. The concept of eddy diffusivities defined by equations (6) is introduced under the condition that Reynolds stress $\overline{uv}$ or turbulent heat flux $\overline{vt}$ has a close relation with the mean velocity gradient or the mean temperature gradient. In the natural convection boundary layer, as shown in Fig. 7, Reynolds stress $\overline{uv}$ takes a very small value of almost zero in the wall vicinity of a large mean velocity gradient and has a positive value at the maximum velocity location ($\partial U/\partial y = 0$) without $\overline{uv}$ becoming zero. Therefore, the distribution of eddy diffusivity for momentum discontinues at the maximum velocity location so that ε_m has a negative value in the wall side and a positive value in the outer layer side. On the other hand, eddy diffusivity for heat shows a continuous profile. In consequence, the distribution of turbulent Prandtl number defined as a ratio of eddy diffusivities for momentum and heat also discontinues at the maximum velocity location. The significance of introducing eddy diffusivities and turbulent Prandtl number is poor for the inner layer from the wall to the maximum velocity location. In the comparatively wide region $y^+ \geqslant 100$ of the outer layer, eddy diffusivities for momentum and heat have a similar profile and so the turbulent Prandtl number takes a value of about unity. Although the measurement of turbulent Prandtl number by Smith [2] is also shown in Fig. 11 by the broken line, the trend is markedly different from the present result. Miyamoto and Okayama [4] reported that the turbulent Prandtl number has a peak value ($Pr_t \simeq 2$) at the maximum $\overline{uv}$ location in the outer layer. However, the present

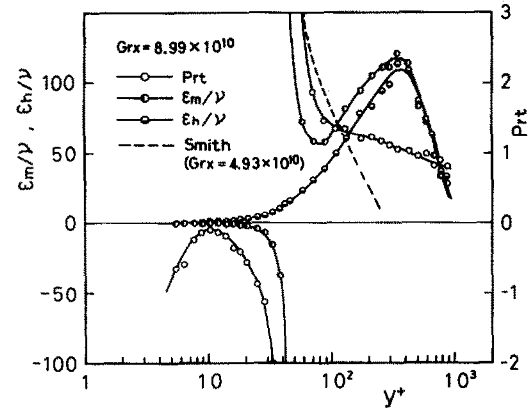

FIG. 11. Eddy diffusivities for momentum and heat and turbulent Prandtl number.

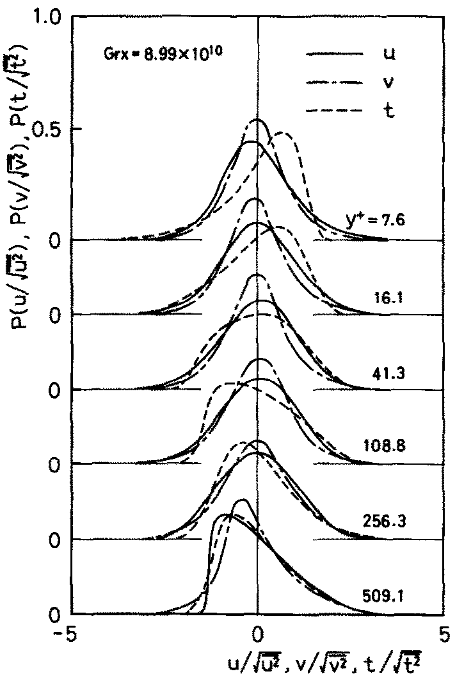

FIG. 12. Probability density functions of velocity and temperature fluctuations.

Pr_t value in the outer layer shows a gradual decrease toward the outer edge of the boundary layer.

Although numerical analyses of natural convection using turbulence models have been developed [2, 22–25], the models of each calculation are based for the most part on the research result for forced convection because of the paucity of reliable measured data for natural convection. It is evident from the present result that some modifications of these models would be needed for the analysis of the turbulent natural convection boundary layer.

6. PROBABILITY DISTRIBUTIONS OF VELOCITY AND TEMPERATURE FLUCTUATIONS

Probability density functions $P(u/\sqrt{\overline{u^2}})$, $P(v/\sqrt{\overline{v^2}})$ and $P(t/\sqrt{\overline{t^2}})$ in the boundary layer, that were systematically investigated to grasp the characteristics of velocity and temperature fluctuations generated in natural convection, are shown in Fig. 12. Since the waveforms of fluctuations observed near the wall are intermittent due to the large viscous effect (Fig. 3), all probability density functions at $y^+ = 7.6$ have peaked distributions. Especially, the distribution for v is most peaked. Although the distribution for u is slightly skewed toward positive, that for t is largely skewed toward negative, indicating that low-enthalpy fluid invades near the wall intermittently. With increasing y^+ (i.e. at $y^+ = 16.1$), the distribution for u becomes very close to a Gaussian distribution but v has a very peaked distribution. The distribution for t is still skewed toward negative. Because the intensity of the temperature fluctuation becomes maximum at this location, the probability distribution for t with a long tail toward the negative side indicates that fluctuating temperature close to ambient fluid temperature already appears near the wall. At $y^+ = 41.3$, corresponding to the vicinity of the maximum velocity location, u still has a Gaussian distribution and v a peaked distribution. The distribution for t becomes flat and skewed toward positive in this location. The distribution for t becomes increasingly skewed toward positive at $y^+ = 108.8$ in the outer layer, while no variation is seen for velocity fluctuations. At $y^+ = 256.3$, where the velocity fluctuations become maximum, u takes a distribution close to Gaussian and v has a peaked distribution, as ever; however, the distribution for t becomes peaked and skewed toward positive, obviously showing the intermittent characteristic of the outer layer. As the outer edge of the boundary layer is approached (i.e. at $y^+ = 509.1$), all distributions become peaked and skewed toward positive.

To express the characteristics of probability density functions quantitatively, the skewness factors $S(u)$, $S(v)$ and $S(t)$ and the flatness factors $F(u)$, $F(v)$ and $F(t)$, which are third- and fourth-order moments of velocity and temperature fluctuations, are presented in Fig. 13. In the case that the probability density function is Gaussian, the skewness factor and the flatness factor take 0 and 3, respectively. These factors vary in the boundary layer according to the probability distributions shown in Fig. 12. The skewness and flatness factors for u and t fluctuations have been reported by Smith [2] and Miyamoto and Okayama [4]. Their results almost agree with the present ones, quantitatively speaking. The probability distribution for the v fluctuation is clarified for the first time in the present study. The characteristic appears in that the probability distribution is rather peaked and the flatness factor $F(v)$ becomes greater than $F(u)$ over the entire boundary layer region, while the u fluctuation has a distribution close to Gaussian in a wide range of the boundary layer. $F(v)$ becomes very large in the

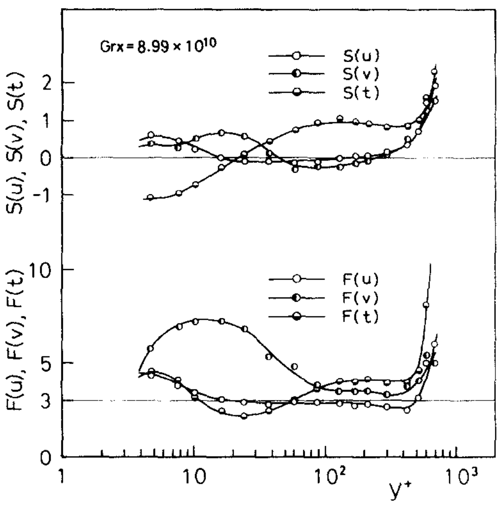

Fig. 13. Skewness and flatness factors of velocity and temperature fluctuations.

inner layer from the wall to the maximum velocity location, and so the v fluctuation shows intermittent behavior.

It is found that the probability distributions of streamwise velocity and temperature fluctuations in a thermal plume measured by George *et al.* [26] correspond comparatively well to those for u and t in the region from the maximum velocity location to the outer layer of the natural convection boundary layer.

7. CONCLUSIONS

Turbulent quantities in the velocity and thermal fields of the turbulent natural convection boundary layer in air along a vertical plate were investigated over the wide region from the near wall to the outer edge of the boundary layer with the probe comprising two V-shaped hot wires and a cold wire.

The results of the present study may be summarized as follows.

(1) The directly measured results of Reynolds stress $\overline{uv}$ and turbulent heat flux $\overline{vt}$ agree very well with the indirectly obtained results estimated by integrating momentum and thermal energy equations with mean velocity and mean temperature measurements. Thus, it is confirmed that the V-shaped hot-wire technique is sufficiently useful and the measurements are more reliable quantitatively than the existing LDV measurements.

(2) Although it is generally conceived that Reynolds stress has a close relation with the mean velocity gradient and becomes negative in the wall vicinity of $\partial U/\partial y > 0$, Reynolds stress near the wall in the natural convection boundary layer, generated by the energy input from the heated surface, is not necessarily negative but becomes almost zero. The maximum Reynolds stress occurs in the outer layer, and the location coincides with that at which both intensities $\sqrt{\overline{u^2}}$ and $\sqrt{\overline{v^2}}$ becomes maximum.

(3) Turbulent heat flux $\overline{vt}$ shows a lower profile in comparision with $\overline{ut}$ in most of the boundary layer, although it becomes slightly larger than $\overline{ut}$ near the wall. The $\overline{vt}$ distribution corresponds rather well to the measured result for forced convection.

(4) For the inner layer from the wall to the maximum velocity location of natural convection, it is difficult to introduce the concept of eddy diffusivity or turbulent Prandtl number in the same manner as forced convection. Therefore, a consideration in this respect is necessary in order to advance the theoretical study.

(5) The probability distribution for v, clarified for the first time in the present study, is fairly peaked in the region from the wall vicinity to the maximum velocity location, while u has a distribution close to Gaussian in most of the boundary layer.

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MESURES DE TURBULENCE DANS UNE COUCHE LIMITE DE CONVECTION NATURELLE LE LONG D'UN PLAN VERTICAL

Résumé—On étudie à l'aide de la technique du fil chaud en V une couche limite turbulente de convection naturelle dont la structure demeure un sujet de conjecture pour qui veut des mesures exploitables. Le lien entre la tension de Reynolds et le flux de chaleur est vérifié par un accord excellent avec les mesures indirectes estimées par intégration des équations de quantité de mouvement et d'énergie et couplées aux vitesses et températures moyennes mesurées. Les grandeurs turbulentes clarifiées quantitativement dans cette étude montrent que la couche limite de convection naturelle a une structure turbulente unique rarement vue dans d'autres couches limites turbulentes.

MESSUNGEN IN DER TURBULENTEN GRENZSCHICHT BEI NATÜRLICHER KONVEKTION AN EINER SENKRECHTEN EBENEN PLATTE

Zusammenfassung—Eine turbulente Grenzschicht bei natürlicher Konvektion, deren Struktur aus Mangel an zuverlässigen Messungen eine Frage von Vermutungen bleibt, wird mit einer Methode mit V-förmigem Hitzdraht untersucht. Die Zuverlässigkeit der Messung der Reynolds-Schubspannung und der Wärmestromdichte wird durch die exzellente Übereinstimmung mit den indirekten Messungen, die durch Integration der Bewegungs- und Energiegleichungen mit der gemessenen mittleren Geschwindigkeit und mittleren Temperatur bestimmt wurden, verifiziert. Die in dieser Arbeit quantitativ bestimmten turbulenten Größen zeigen, daß die Grenzschicht bei natürlicher Konvektion eine einzigartige turbulente Struktur besitzt, die in anderen turbulenten Grenzschichten selten anzutreffen ist.

ИЗМЕРЕНИЕ ТУРБУЛЕНТНОСТИ ЕСТЕСТВЕННОКОНВЕКТИВНОГО ТЕЧЕНИЯ В ПОГРАНИЧНОМ СЛОЕ НА ВЕРТИКАЛЬНОЙ ПЛОСКОЙ ПЛАСТИНЕ

Аннотация—С помощью термоанемометра с V-образным датчиком исследуется турбулентное естественноконвективное течение в пограничном слое, представление о структуре которого базируется пока на догадках из-за отсутствия надежных измерений. Достоверность экспериментальных данных, полученных для напряжения Рейнольдса и турбулентного теплового потока, подтверждается хорошим совпадением с результатами косвенных измерений, оценка которых производилась путем интегрирования уравнений количества движения и конвективного теплопереноса с учетом экспериментальных данных для средних значений скорости и температуры. Полученные в данной работе численные значения турбулентных величин свидетельствуют о том, что естественноконвективный пограничный слой имеет уникальную структуру, редко наблюдаемую в других видах турбулентных пограничных слоев.